Optimum Design of Composite Honeycomb Sandwich Panels Subjected to Uniaxial Compression

Jack R. Vinson*
University of Delaware, Newark, Delaware

Sandwich construction provides a very lightweight structural configuration for many load conditions. The use of composite materials with their high stiffness, high strength, and anisotropy makes sandwich construction even more competitive for many applications. It is very desirable to design these structures for minimum weight to insure their most effective use. Closed-form analytical solutions are presented herein for the analysis and design of minimum weight, composite material hex-cell and square cell honeycomb core sandwich and panels subjected to in-plane uniaxial compressive loads. These methods account for overstressing, overall buckling, core shear instability, face wrinkling, and monocell buckling. The optimum face thickness, core depth, cell wall thickness, and cell size are analytically determined. The methods insure minimum weight, as well as provide methods to compare various material systems, compare honeycomb sandwich construction with other panel architectures, and assess the weight penalties associated with using nonoptimum honeycomb sandwich constructions. A comparison of various polymer, metal, and ceramic matrix composite materials is made by way of example.

Nomenclature

A = constant defined Eq. (4)
a = panel dimension in the load direction
B_i = constants defined Eq. (5), $i = 1\text{-}3$
b = panel dimension in the unloaded directions
C_i = constants defined by Eqs. (7-10), $i = 1\text{-}4$
D_i = flexural stiffness of the sandwich panel in the i direction, $E_{fi} h_c^2 t_f / 2(1 - \nu_{xy}\nu_{yx})$, $i = x, y$
d = diameter of an inscribed circle in a honeycomb cell, see Fig. 2.
E'_c = effective core transverse stiffness, see Eq. (19)
E_{fi} = modulus of elasticity of the face material in the i direction, $i = x, y$
E_{fT} = tangent modulus of the face material in the i direction
FM = factor of merit of the face material, $= E_{fx}^{\frac{3}{8}} E_{fy}^{\frac{1}{8}} / \rho_f (1 - \nu_{xy}\nu_{yx}) 1/2$
G'_{ci} = effective transverse core shear stiffness in the i direction ($i = x, y$), see Eqs. (18) and (19)
H_x = extensional stiffness of the sandwich panel in the load direction, $= 2 t_f E_{fx}$
h_c = sandwich core depth
K = buckling constant defined by Eq. (3)
k_1 = boundary condition factor
k_i = core geometric factors, see Eqs. (18) and (19), $i = 2\text{-}4$
N_x = applied compressive load to the sandwich panel per unit width in the unloaded direction
r = core orthotropy factor, ($= V_y/V_x = G'_{ci}/G'_{cy} = k_3/k_4$)
t_f = face thickness
U_{iz} = transverse core shear stiffness, $= G'_{ci} h_c$, $i = x, y$
V_f = volume fraction
V_i = ratio of face-to-core stiffness ratio, see Eq. (6), $i = x, y$
W = weight of sandwich panel per unit planform area
W_{ad} = weight of adhesive per unit planform area
η = plasticity reduction factor
ν_{ij} = Poisson's ratio defined by $E_x \nu_{yx} = E_y \nu_{xy}$
ρ_i = weight density of the i material, $i = c, f$
ρ'_c = effective weight density of the core material, see Eq. (18)
σ_{cr} = critical buckling stress
σ_0 = optimum buckling stress, see Eq. (30)
σ_x = applied stress in the load direction, see Eq. (1)

Received July 31, 1985. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1986. All rights reserved.

*The H. Fletcher Brown Professor of Mechanical and Aerospace Engineering. Associate Fellow AIAA.

Introduction

MANY studies have been made during the last 40 years to structurally optimize flat sandwich panels. In 1944, Marguerre[1] investigated the optimum buckling strength of longitudinally stressed flexibly supported plates consisting of two metal faces and a light filler. Only two buckling equations were considered: buckling of the plate as a whole, accounting for the shear elasticity of the filler; and the "crumpling" of the metal face sheets. Design charts were developed for the elastic range as well as the inelastic range and, from these diagrams, the face thickness and the weight and core thickness could be read directly.

In 1949, Flugge[1] studied the structural optimization of sandwich panels in which he presented nomograms for the solution of the following problems: 1) the geometric dimensions and the core properties for a given compression load and minimum weight, 2) the geometrical dimensions and core properties for a given weight and maximum compressive load, and 3) the ultimate strength of a given sandwich. In all cases studied, the materials were isotropic and the following cases of failure were considered: overall buckling, crinkling (wrinkling), and the elastic limit of the faces. The unloaded edges were free or simply supported. Flugge published another paper on the subject in 1952.

In 1950, Bijlaard[1] approached the subject of sandwich optimization by considering plates with given weight per unit surface area and computing the ratio of the elastic moduli of core and faces that lead to a maximum buckling load. He carried out the optimization for a given ratio between thicknesses of the sandwich plate. Only isotropic materials were considered. An abridgement of this publication appeared in the *Proceedings* of the 1st U.S National Congress of Applied Mechanics in 1951.

Gerard[1] briefly discusses sandwich plate optimization in one chapter of his book, *Minimum Weight Analysis of Compression Structures*.

Kaechele,[1] in 1957, wrote on the minimum-weight design of sandwich panels. He presented a method for determining the optimum configuration of flat, simply supported sandwich panels under uniaxial compression, when the load, width, and stress-strain curves for the face material are given. He also invoked side conditions on the maximum allowable face stress (or strain) and on core strength or density. He applied the method to both hexagonal cell and square cell honeycomb core panels and discussed elevated temperature effects briefly.

In 1960, Heath[1] published a paper on the correlation and extension of existing theory of flat panels of sandwich construction subjected to lengthwise compression. The second part of this paper was concerned with the optimum design of flat sandwich panels.

In general, these studies failed to provide a true optimum for several reasons. It has been long recognized that a truly optimum structure, having several independent failure modes under a specific type of loading, will fail simultaneously in each failure mode for a given applied load. If this is not the case, it means that one or more modes of failure occurs at a stress higher than other modes. This in turn means there exists material in the structure (which has weight) that is not being stressed or strained sufficiently high for it to be used efficiently. Thus, the weight of that structure can be reduced further by removing material until all modes of failure occur simultaneously or by shifting the material to increase the properties associated with weaker failure modes retaining the same weight.

Also, the previous optimization studies did not equate the number of failure modes or other relevant equations to the number of design degrees of freedom in the structure, in order that specific unique values of all design parameters can be determined for the optimum (minimum weight) structure. Only by equating the number of pertinent equations to the number of variables can there be a unique minimum weight solution.

More recently, optimization studies of composite materials panels have been performed. In 1975, Housner and Stein[4] calculated the optimum fiber direction for graphite-epoxy sandwich panels under axial compression, assuming all ply angles to be identical. In 1979, Hirano presented a method to design laminated plates of orthotropic layers under uniaxial and biaxial compression. He extended that work in 1980.[5,6]

General

Consider a rectangular sandwich panel of length a (the load direction), width b, face thickness t_f, core depth h_c, core cell wall thickness t_c, and diameter d of a circle inscribe as shown in Fig. 1 for a hexagonal core and Fig. 2 for the hex-cell honeycomb core. A sketch analogous to Fig. 2 could be drawn for the square cell honeycomb. It is assumed for this study that the core is composed of an isotropic material of shear stiffness G_c and modulus of elasticity E_c. If the core is orthotropic, the properties normal to the plane of the panel for E_c and G_c are used. Consider the faces to be composed of identical composite materials that are balanced about their own plane of symmetry with no unwanted couplings, i.e., $B_{ij} = 0$ and all 16 and 26 terms = 0. It is assumed that the in-plane load in the x direction is uniform and has the value of N_x (load/unit width). Also, it is assumed that the in-plane loads are resisted only by the faces, not the honeycomb core. Therefore, the applied compressive face stress in the load direction is written as

$$\sigma_x = N_x / 2t_f \tag{1}$$

For an applied load N_x, the face stress σ_x is, of course, restricted to some prescribed maximum value to prevent overstressing.

Buckling Analysis

The honeycomb sandwich panel shown in Fig. 1 can be overstressed according to Eq. (1), but can also buckle in one of several modes, any one of which will render the panel to be useless for this study. These modes of buckling are overall instability, core shear instability, face wrinkling, and face dimpling (monocell buckling). These are discussed in turn.

Overall Instability

The equation to use for the overall buckling of the subject panel that includes the effects of transverse shear deformation is given by

$$\sigma_{cr} = \frac{E_{fx}\pi^2\sqrt{D_x D_y}\,K}{b^2 H_x} \tag{2}$$

In this equation, the coefficient K is given by

$$K = \frac{B_1C_1 + 2B_2C_2 + \dfrac{C_3}{B_1} + A\left[\dfrac{V_y}{C_4} + V_x\right]}{1 + (B_1C_1 + B_3C_2)\dfrac{V_y}{C_4} + \left(\dfrac{C_3}{B_1} + B_3C_2\right)V_x + \dfrac{V_yV_xA}{C_4}} \tag{3}$$

where

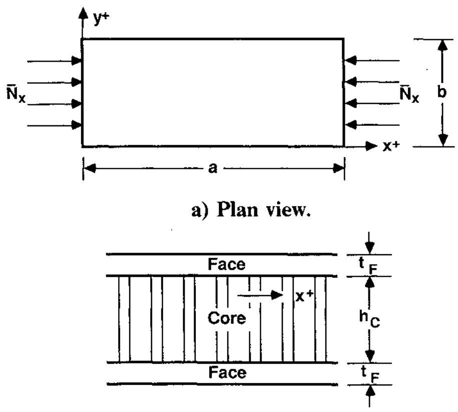
$$A = C_1C_3 - B_2C_2^2 + B_3C_2\left(B_1C_1 + 2B_2C_2 + \frac{C_3}{B_1}\right) \tag{4}$$

$$B_1 = \sqrt{\frac{D_y}{D_x}}\,,\quad B_2 = \frac{D_y\nu_{xy}}{\sqrt{D_xD_y}} = 2B_3 + B_1\nu_{xy}\,,\quad B_3 = \frac{D_{xy}}{\sqrt{D_xD_y}} \tag{5}$$

$$V_y = \frac{\pi^2}{b^2}\frac{\sqrt{D_xD_y}}{U_{yz}}\,,\quad U_{yz} = G'_{cy}h_c$$

$$V_x = \frac{\pi^2}{b^2}\frac{\sqrt{D_xD_y}}{U_{xz}}\,,\quad U_{xz} = G'_{cx}h_c \tag{6}$$

The values C_1 through C_4 in Eqs. (3) and (4) are associated with the boundary conditions and are listed as follows for completeness. n is the number of waves in the direction of compressive loading.



a) Plan view.

b) Side view.

Fig. 1 Sandwich panel.

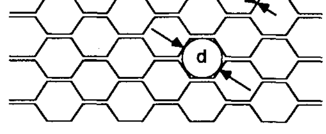


Fig. 2 Honeycomb core in planview.

1) All edges simply supported

$$C_1 = C_4 = \frac{a^2}{n^2 b^2}, \qquad C_2 = 1, \qquad C_3 = \frac{n^2 b^2}{a^2} \tag{7}$$

2) Loaded edges simply supported, other edges clamped

$$C_1 = \frac{16}{3}\frac{a^2}{n^2 b^2}, \qquad C_2 = \frac{4}{3}, \qquad C_3 = \frac{n^2 b^2}{a^2}, \qquad C_4 = \frac{4}{3}\frac{a^2}{n^2 b^2} \tag{8}$$

3) Loaded edges clamped, other edges simply supported

$$C_1 = C_4 = \frac{3}{4}\frac{a^2}{b^2} \quad \text{for } n = 1$$

$$C_1 = C_4 = \frac{1}{(n^2+1)}\frac{a^2}{b^2} \quad \text{for } n \geq 2$$

$$C_2 = 1, \; C_3 = \frac{n^4 + 6n^2 + 1}{(n^2+1)}\frac{b^2}{a^2} \tag{9}$$

4) All edges clamped

$$C_1 = 4C_4 = \frac{4a^2}{b^2} \quad \text{for } n = 1$$

$$C_1 = 4C_4 = \frac{16}{3(n^2+1)}\frac{a^2}{b^2} \quad \text{for } n \geq 2$$

$$C_2 = \frac{4}{3}, \quad C_3 = \frac{(n^4 + 6n^2 + 1)}{n^2+1}\frac{b^2}{a^2} \tag{10}$$

Equation (2) can be reduced to the following expression:

$$\sigma_{\rm cr} = \frac{\pi^2 \sqrt{E_{fx} E_{fy}}}{4(1 - \nu_{xy}\nu_{yx})}\frac{h_c^2}{b^2} K \tag{11}$$

Core Shear Instability

Referring to the expressions for overall stability, if the value of V_x is increased through increasing the panel bending stiffnesses (D_x and D_y) or decreasing the core transverse shear stiffness (U_{xz}), the values of K in Eq. (3) are decreased. There exists a value of V_x that causes K to equal $1/V_x$. This value depends both on the boundary conditions and the effective shear moduli of the core (G'_{cx}, G'_{cy}). At this particular value of $K = 1/V_x$, K is independent of the length-to-width ratio a/b and n is infinite. For values of V_x greater than this value, $K_M = 1/V_x$, which is true for a great number of practical sandwich panels. Under these conditions, the critical stress can be written as

$$\sigma_{\rm cr} = E_{fx} U_{xz} / H_x \tag{12}$$

Core shear instability is illustrated in Fig. 3.

Table 1 Boundary condition factors k_i for various edge conditions

Boundary condition	k_1
All edges simply supported	1
Loaded edges simply supported, other edges clamped	3/4
Loaded edges clamped, other edges simply supported	1
All edges clamped	3/4

This value of critical stress is called the core shear instability stress and cannot be exceeded for any given sandwich construction. It is seen that this stress is independent of the panel length, width, and boundary conditions.

The particular value of V_x at which $K_M = 1/V_x$, at which the critical stresses given by Eqs. (1) and (12) are equal, as stated before, is given in Eq. (13) and is dependent upon the boundary conditions and the effective shear modulus of the core, G'_{cx} and G'_{cy}. The values for the boundary condition factors k_1 are listed in Table 1.

$$V_x = k_1 B_1 r \tag{13}$$

Face Wrinkling Instability

Wrinkling occurs across many cells of the honeycomb core and, under the loading conditions described here, extends across the width of the plate, but is localized in the direction of the axial load; that is, the wrinkle is essentially a short wavelength buckle, as shown in Fig. 4.

Heath[1] derived an expression for this mode of instability for the case of isotropic materials as

$$\sigma_{\rm cr} = \left[\frac{2}{3}\frac{t_f}{h_c}\frac{E'_c E_f}{(1-\nu^2)}\right]^{\frac{1}{2}} \tag{14}$$

Heath defined E'_c incorrectly in his paper, but Hemp[1] clarified the point. The face wrinkling stability equation for isotropic faces given by Eq. 14 can be modified analogously to the following expression for anisotropic materials:

$$\sigma_{\rm cr} = \left[\frac{2}{3}\frac{t_f}{h_c}\frac{E'_c\sqrt{E_{fx}E_{fy}}}{(1-\nu_{xy}\nu_{yx})}\right]^{\frac{1}{2}} \tag{15}$$

Monocell Buckling or Face Dimpling

In honeycomb core sandwiches, a fourth type of instability occurs because the faces over one cell can buckle as a small plate supported by the cell walls. Methods of analysis developed at the Forest Products Laboratory used an empirical equation having the form of the plate buckling equation with the numerical coefficient determined by empirical means. The result for an isotropic face material is

$$\sigma_{\rm cr} = \frac{2E_{fT}}{(1-\nu^2)}\left(\frac{t_f}{d}\right)^2 \tag{16}$$

For anisotropic faces, the expression can be written as

$$\sigma_{\rm cr} = \frac{2(E_{fx_T}E_{fy})^{\frac{1}{2}}}{(1-\nu_{xy}\nu_{yx})}\left(\frac{t_f}{d}\right)^2 \tag{17}$$

Core Properties

Mechanical properties of honeycomb core used in the previous equations are called "effective" and are designated with a prime, because they are properties associated with the core acting as a homogeneous material having these "effective" properties. They are functions of the core materials E_c, G_{cx},

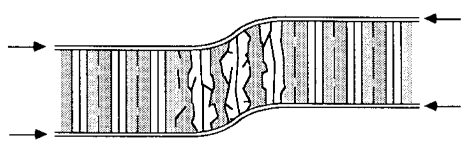


Fig. 3 Core shear instability.

and G_{cy}, the core wall thickness t_c, and the cell size d. To truly optimize the structure for minimum weight, it is advantageous to relate the apparent properties back to fundamental geometry and material properties.

For hexagonal cell honeycomb core having some double walls as shown in Fig. 2, the properties were developed by Kaechele.[1]

For hexagonal cell construction of Fig. 2 as well as other types of honeycomb core, the effective core properties can be related to the geometry and actual material properties as follows:

$$\rho_c' = k_2(t_c/d)\rho_c$$

$$G_{cx}' = k_3(t_c/d)G_c \tag{18}$$

$$G_{cy}' = k_4(t_c/d)G_c$$

$$E_c' = k_2(t_c/d)E_c \tag{19}$$

It is seen from Eqs. (5), (18), and (19) that

$$r = k_3/k_4 \tag{20}$$

For the construction of Fig. 2 as well as the square cell honeycomb, the values of k_2, k_3, and k_4 are given in Table 2, according to Kaechele[1] and MIL HDBK-23.

For other honeycomb configurations, the values of these constants can be easily derived using the methods of Kaechele.[1]

Plasticity Effects

The extension of elastic buckling theory to account for the buckling of structures at stresses above the proportional limit of the material has been studied widely. Many investigations have used the elastic equations, where Young's modulus E has been multiplied by a plasticity reduction factor η. However, there is considerable difference of opinion about a correct form for η. These expressions range in complexity and it is not at all clear which expression has more merit. For this structural optimization, if the compressive stress strain curve of the face material has a proportional limit, then for stresses above the proportional limit, all values of E_{fx} can be replaced by $\bar{E}_{fx}$, where

$$\bar{E}_{fx} = \eta E_{fx} \tag{21}$$

It is unlikely that for the uniaxial compression in the x direction that the stresses in the y direction will cause deviation from the elastic value E_y, but this could also be modified in that case.

Structural Optimization

For a honeycomb sandwich panel of a given length, width, and applied load, the variables with which to optimize the structure are the face thickness t_f, core depth h_c, core wall thickness t_c, cell size (denoted by d), as well as the face and core materials. In the following, it is assumed that the materials are given, either because they are specified by other considerations or, alternatively, that the optimizations are performed for each of several materials and the results compared on a basis of weight as a function of the load index to ultimately select the best materials.

The minimum weight structure will occur when the critical stresses for each mode of instability given in Eqs. (11), (12), (15), and (17–24) are equal, with an upper bound on the stress being some maximum set value, probably a function of the ultimate face stress. To restate the philosophy, the sandwich panel is rendered useless if any of the four modes of buckling occurs. Hence, if one mode has a critical stress higher than others, material can be shifted, with no change of weight, until all buckling modes occur simultaneously at a critical stress σ_0. It is seen that under these conditions the following equation can be substituted for Eq. (11):

$$V_x = k_1 B_1 r \tag{22}$$

since for this equality, the critical stress for the overall stability equals the critical stress for the core shear instability.

To obtain a weight per unit area of the honeycomb sandwich structure, the following may be used:

$$W = 2t_f\rho_F + \rho_c' h_c + W_{ad} \tag{23}$$

The weight of the adhesive or other joining material cannot be easily related to the variables discussed earlier and are dependent upon the material, method of joining, fabrication techniques, and skill and temperament of the personnel. Since in many cases, this is a small fraction of the weight and, because of the factors involved, it will not be specified further and need not be accounted for in the comparisons to select the optimum geometry and materials. However, care should be used to include it when comparing structures employing no adhesive or with other types of construction.

Substituting in the value of the core properties, the final set of relations to utilize in the optimization are given as

$$\sigma_{cr} = \frac{k_3}{2}\left(\frac{t_c}{d}\right)\left(\frac{h_c}{t_f}\right)G_c \tag{24}$$

Table 2 Values of k_2, k_3, and k_4 for various honeycomb constructions

Type of construction	k_2	k_3	k_4
Hex-cell (Kaechele)	8/3	5/3	1
Hex-cell (MIL HDBK-23)	8/3	4/3	8/15
Square cell (Kaechele)	2	1	1
Square cell (MIL HDBK-23)	2	1	1

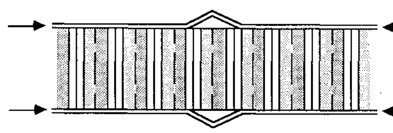


Fig. 4 Face wrinkling instability.

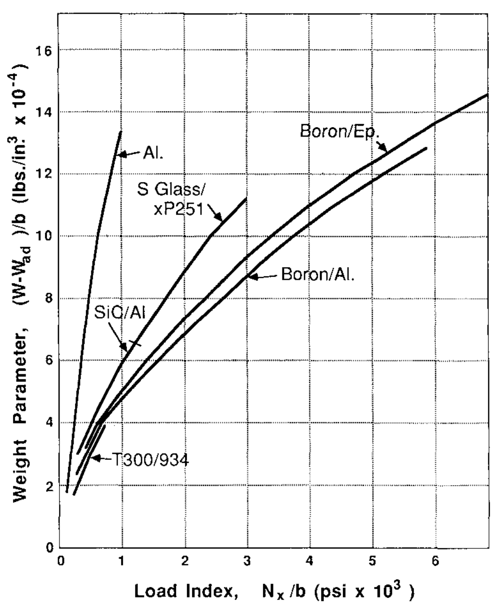


Fig. 5 Materials comparison.

Table 3 Materials comparison

Material	Configuration	V_F, %	E_x, msi	E_y, msi	ρ, lb/in.^{3}	σ_{ult}, ksi[a]	FM[b]	Ref.
T300/934	Unidir.	60	23.69	1.7	0.0555	105	63.09	7
C/epoxy	Cross-ply	58	12.04	12.04	0.0555	55.1(T)	62.50	8
T300/5208	Unidir.	72	22.2	1.58	0.0555	110	61.01	7
T300/SP286	Unidir.	60	21.9	1.53	0.0555	164	60.46	7
B/2024Al	—	49	33.93	22.04	0.0916	220(T)	60.24	9
B/6061Al	—	49	33.79	22.04	0.0916	513.3	60.14	9
B(B_4C)/Al	—	49	34.95	18.85	0.0918	210.3(T)	59.62	9
B/5052Al	—	49	33.35	21.03	0.0916	168.2(T)	59.50	9
B/1100Al	—	49	33.35	19.72	0.0916	323.3	59.03	9
B/3002Al	—	49	32.48	20.74	0.0916	365.4	58.81	9
B/6061	Unidir.	50	32.0	20.00	0.0915	250.0	58.33	8
AS/3501	—	67	20.2	1.30	0.0555	209.9	57.47	7
AS1/3501-6	—	—	18.85	1.52	0.0555	246.0	57.10	10
B/Al	—	48	30.02	21.03	0.0918	221.9(T)	57.09	9
Borsic/Al	—	45	31.03	20.16	0.0926	190.0(T)	57.03	9
B/epoxy	—	67	31.18	3.05	0.0740	468.4	56.43	9
B/epoxy	—	67	30.3	2.80	0.0740	362.6	55.24	7
B/epoxy	Cross-ply	60	15.37	15.37	0.0721	55.1(T)	54.37	8
T300/2500	—	—	17.55	1.17	0.0555	—	53.80	11
$\alpha Al_2O_3/M_g$	Cont. fiber	50	30.02	15.08	0.1009	275.5	49.89	9
Kevlar49/epoxy	Cross-ply	60	5.80	5.80	0.0505	—	47.72	8
SiC/6061Al	—	48	31.63	18.06	0.1122		46.74	8
S glass/xp-251		67	8.29	2.92	0.0555	170.0	45.53	7
γAl_2O_3/Al	Cont. fiber	50	21.75	15.95	0.1045	203.0	42.91	9
γAl_2O_3/Al-5Cu	Cont.fiber	50	21.75	14.50	0.1045	319.0	42.41	9
SiC/Ti	Woven, ISO	39.5	28.70	28.70	0.1432	228(T)	37.41	8
αAl_2O_2/Al	Cont. fiber	50	31.9	20.16	0.1172	406	36.14	9
SiC/Ti-6Al-4V	Woven, ISO	35	27.0	27.0	0.1492	245(T)	34.83	8
2024 Al	ISO	—	10.88	10.88	0.0973	—	33.88	9
Titanium	ISO	—	17.4	17.4	0.1600	145	26.07	9
Titanium	ISO	—	16.5	16.5	0.1636	170(T)	24.98	8
E glass/epoxy	Cross-ply	57	3.12	3.12	0.0710	82.0	24.87	8
Welton 80 steel	ISO	—	30.45	30.45	0.283	126.9	19.50	9

[a]Compressive strength: (T) denotes tensile properties available only. [b]Units of FM are 10^3 in.^{2}/lb$^{1/2}$.

$$\sigma_{cr} = \left[\frac{2}{3} \frac{k_2}{(1 - \nu_{xy}\nu_{yx})} \left(\frac{t_f}{h_c} \right) \left(\frac{t_c}{d} \right) E_c \sqrt{E_{fx} E_{fy}} \right]^{\frac{1}{2}} \tag{25}$$

$$\sigma_{cr} = \frac{2\sqrt{E_{fx_T} E_{fy_T}}}{(1 - \nu_{xy}\nu_{yx})} \left(\frac{t_f}{d} \right)^2 \tag{26}$$

From the expression $V_x = k_1 B_1 r$, we obtain

$$\frac{\pi^2}{2k_3(1 - \nu_{xy}\nu_{yx})} \frac{t_f h_c d}{b^2 t_c} \frac{E_{fx}}{G_c} = k_1 r \tag{27}$$

$$N_x = 2 t_f \sigma_x \tag{28}$$

$$W = 2\rho_f t_f + k_2 \left(\frac{t_c}{d} \right) h_c \rho_c + W_{\mathrm{ad}} \tag{29}$$

Equations (24–28) yield the *universal relationship for optimum honeycomb sandwich panels with orthotropic facings and core under uniaxial compression*,

$$\left(\frac{k_2}{k_3 k_1 r} \right)^{\frac{1}{2}} \left(\frac{N_x}{b} \right) = \frac{2\sqrt{3}(1 - \nu_{xy}\nu_{yx})}{\pi} \left(\frac{G_c}{E_c} \right)^{\frac{1}{2}} \frac{\sigma_0^2}{E_{fx}^{\frac{3}{4}} E_{fy}^{\frac{1}{4}}} \tag{30}$$

It must be noted that, in Eq. (30), σ_0 must be limited to some predetermined allowable stress; hence, there is an upper bound on the load that can be carried by the panel for a given material system.

Therefore, it is seen that, when given a set of boundary conditions (denoted by k_1) and any type of honeycomb core construction (denoted by k_2, k_3, and r), the universal relationship relates the load index N_x/b to material properties only; it is independent of geometric variables and it establishes a unique optimum buckling stress σ_0 for each value of load index.

The other key optimization relationships are found from Eqs. (24–28) and (30) to be

$$\left(\frac{k_2}{k_3} \right)^{\frac{1}{2}} \left(\frac{t_f}{h_c} \right) = \frac{1}{2} \left[3(1 - \nu_{xy}\nu_{yx}) \frac{G_c}{E_c} \frac{\sigma_0}{\sqrt{E_{fx} E_{fy}}} \right]^{\frac{1}{2}} \tag{31}$$

$$(k_2 k_3)^{\frac{1}{2}} \left(\frac{t_c}{d} \right) = \left[3(1 - \nu_{xy}\nu_{yx}) \frac{\sigma_0^3}{G_c E_c \sqrt{E_{fx} E_{fy}}} \right]^{\frac{1}{2}} \tag{32}$$

$$\frac{t_f}{d} = \left[\frac{1 - \nu_{xy}\nu_{yx}}{2} \frac{\sigma_0}{\sqrt{E_{fx_T} E_{fy}}} \right]^{\frac{1}{2}} \tag{33}$$

Explicit values of the uniquely optimum geometric variables as functions of the load index N_x/b are determined to be

$$h_c = b \left(\frac{2}{\pi} \right)^{\frac{3}{4}} \left(\frac{k_2}{3k_3} \right)^{\frac{1}{8}} (1 - \nu_{xy}\nu_{yx})^{\frac{1}{4}} (k_1 r)^{\frac{3}{8}} \left(\frac{E_c}{G_c} \right)^{\frac{1}{8}} \frac{E_{fy}^{\frac{1}{16}}}{E_{fx}^{\frac{5}{16}}} \left(\frac{N_x}{b} \right)^{\frac{1}{4}} \tag{34}$$

$$d = \frac{b 2^{\frac{1}{4}}}{\pi^{\frac{3}{4}}} \left(\frac{3k_3 k_1 r}{k_2} \right)^{\frac{3}{8}} (1 - \nu_{xy}\nu_{yx})^{\frac{1}{4}} \left(\frac{G_c}{E_c} \right)^{\frac{3}{8}} \frac{E_{fx_T}^{\frac{1}{4}} E_{fy}^{\frac{1}{16}}}{E_{fx}^{\frac{9}{16}}} \left(\frac{N_x}{b} \right)^{\frac{1}{4}} \tag{35}$$

$$t_c = b \left[\frac{3}{2k_2 k_3} \frac{1}{G_c E_c} \sqrt{\frac{E_{fx_T}}{E_{fx}}} \right]^{\frac{1}{2}} \left(\frac{N_x}{b} \right) \tag{36}$$

$$t_f = b\left(\frac{3k_3k_1r}{k_2}\right)^{\frac{1}{4}}\left[\frac{1-\nu_{xy}\nu_{yx}}{2\pi}\right]^{\frac{1}{2}}\left(\frac{G_c}{E_c}\right)^{\frac{1}{4}}\frac{(N_x/b)^{\frac{1}{2}}}{E_{fx}^{\frac{3}{8}}E_{fy}^{\frac{1}{8}}} \tag{37}$$

The explicit values of the optimum geometric variables in terms of optimum critical stresses σ_0 are written as

$$h_c = b\left(\frac{2}{\pi}\right)(k_1r)^{\frac{1}{2}}(1-\nu_{xy}\nu_{yx})^{\frac{1}{2}}\frac{\sigma_0^{\frac{1}{2}}}{E_{fx}^{\frac{1}{2}}} \tag{38}$$

$$d = \frac{b}{\pi}\left[\frac{6(1-\nu_{xy}\nu_{yx})k_3k_1r}{k_2}\right]^{\frac{1}{2}}\left(\frac{G_c}{E_c}\right)^{\frac{1}{2}}\frac{E_{fx_T}^{\frac{1}{4}}\sigma_0^{\frac{1}{2}}}{E_{fx}^{\frac{3}{4}}} \tag{39}$$

$$t_c = \frac{b3\sqrt{2}}{\pi k_2}(k_1r)^{\frac{1}{2}}(1-\nu_{xy}\nu_{yx})\frac{E_{fx_T}^{\frac{1}{4}}\sigma_0^2}{E_cE_{fx}E_{fy}^{\frac{1}{4}}} \tag{40}$$

$$t_f = b\left(\frac{3k_3k_1r}{k_2}\right)^{\frac{1}{2}}\frac{(1-\nu_{xy}\nu_{yx})}{\pi}\left(\frac{G_c}{E_c}\right)^{\frac{1}{2}}\frac{\sigma_0}{E_{fx}^{\frac{3}{4}}E_{fy}^{\frac{1}{4}}} \tag{41}$$

The expressions for the weight per unit planform area for the optimum panel as a function of optimum stress σ_0 and the load index N_x/b can be found by substituting the above expressions into Eq. (29). The results are

$$W = \left(\frac{3k_3k_1r}{k_2}\right)^{\frac{1}{2}}\frac{2(1-\nu_{xy}\nu_{yx})b}{\pi} \times\left(\frac{G_c}{E_c}\right)^{\frac{1}{2}}\frac{\sigma_0}{E_{fx}^{\frac{3}{4}}E_{fy}^{\frac{1}{4}}}\left[\rho_f+\rho_c\frac{k_2}{k_3}\frac{\sigma_0}{G_c}\right]+W_{\mathrm{ad}} \tag{42}$$

$$W = \left(\frac{3k_3k_1r}{k_2}\right)^{\frac{1}{4}}\left[\frac{2(1-\nu_{xy}\nu_{yx})}{\pi}\right]^{\frac{1}{2}} \times\left(\frac{G_c}{E_c}\right)^{\frac{1}{4}}\frac{b(N_x/b)^{\frac{1}{2}}}{E_{fx}^{\frac{3}{8}}E_{fy}^{\frac{1}{8}}}\left[\rho_f+\rho_c\frac{k_2}{k_3}\frac{\sigma_0}{G_c}\right]+W_{\mathrm{ad}} \tag{43}$$

Several very important and interesting conclusions can be drawn from Eq. (43) about the weight of *optimized* honeycomb sandwich panels under uniaxial compressive loads.

It is seen that in the selection of the materials to use in these optimum panels, the core material with the highest ratio of G_c/ρ_c will result in minimum weight. For the selection of a facing material, the material that will result in the panel of lowest weight is that which has the highest ratio of $E^{\frac{3}{8}}{}_{fx}E^{\frac{1}{8}}{}_{fy}/\rho_f(1-\nu_{xy}\nu_{yx})$ for the particular load index N_x/b. For optimized honeycomb sandwich panel construction, the ratio of core weight to face weight is

$$\frac{W_c}{W_f} = \frac{\rho_c}{\rho_f}\frac{k_2}{k_3}\frac{\sigma_0}{G_c} \tag{44}$$

Therefore, it is obvious that, for a construction employing the same material in both faces and honeycomb core, the great majority of the weight is in the faces. Also note that the ratio of the core weight to face weight for an optimum panel is independent of boundary conditions and the orthotropy factor r. In Eq. (43), it is seen that the panel weight varies as the one-fourth power of the boundary condition factor k_1. For a panel that is simply supported on the unloaded edges, $k_1 = 1$. In a panel that is clamped on the unloaded edges, $k_1 = \frac{3}{4}$. Therefore, the ratio of the weight of an optimum panel simply supported on the unloaded edges to the weight of an optimum panel clamped along the unloaded edges is 1.0745.

This is a slight over-simplification, but it can be concluded that for a given *load index* an optimum honeycomb sandwich panel simply supported on the unloaded edges weighs *no more than* 1.0745 times the weight of the optimum panel with unloaded edges clamped.

This result has major implications. Foremost, it implies that, in almost all cases of the optimization of sandwich panels subjected to uniaxial compression, the optimizations should be conducted for simply supported boundary conditions on all edges. (Note that the boundary conditions on the loaded edges have no effect for this case of loading.) Also,

1) Such an optimization will result in the panel weighing at most 7.45% over an optimum panel whose unloaded edges are clamped. Choosing a simply supported panel is thus conservative as far as all dimensions selected, enabling the panel to have additional structural integrity even when the edges are clamped or partially clamped.

2) In actual construction, the auxiliary structural elements required to make the unloaded edges clamped would possibly offset the potential saving of 7.45% in weight resulting from the clamped boundary condition. In the final analysis, the clamped panel assembly would weigh more.

3) It is virtually impossible to insure a clamped edge; hence, most panel edge conditions are between the conditions of simply supported and clamped. The choice of simple support conditions upon which to optimize is therefore conservative as well as rational.

Comparison of Various Material Systems

For a honeycomb core sandwich panel subjected to a uniaxial inplane compressive load, independent of the boundary condition, the "best" face material is determined by

$$\mathrm{FM} = \frac{E_x^{\frac{3}{8}}E_y^{\frac{1}{8}}}{\rho_f(1-\nu_{xy}\nu_{yx})^{\frac{1}{2}}} \tag{45}$$

It is of interest to use this figure of merit (FM) on some of the material systems of present interest where, because the Poisson's ratios are rarely given tables of mechanical properties for the new materials, it is assumed below that $(1-\nu_{xy}\nu_{yx}) \approx 1$. The values in Table 3 are for 70 F.

It is seen clearly that in this application composite materials are better than any all-metallic construction. It is also clear that graphite/epoxy and boron/aluminum composites clearly result in the minimum weight construction. Even boron/epoxy is quite competitive.

The above comparison does not indicate maximum loads that the panel may carry. To investigate that, knowledge of maximum allowable stresses must be available and Eq. (7) used to determine the maximum load index N_x/b to which the optimum panel may be subjected. Figure 5 illustrates that point, where the maximum allowable stresses are taken to be the ultimate compressive strength and a 2024 aluminum core is arbitrarily chosen. It is seen that T300/934 is the lowest weight material, but limited to a load index of $N_x/b = 719$ psi. For higher load levels, boron/aluminum is the most efficient material to a load index of 5840 psi and boron/epoxy the best to a load index of 8740 psi. Beyond that, other panel configurations are perhaps required, which will be the subject of subsequent publications.

In all of the above, it is assumed the materials are elastic to the ultimate stress. If ductility permits and the stress strain curves known for stresses higher than the yield point, a plasticity reduction factor can be employed.

Finally, it should be remembered that in any laminated plate of any construction where $B_{ij} = (\ )_{16} = (\ )_{26} = 0$, E_x and E_y can always be obtained by utilizing the appropriate stiffness matrix values.

Acknowledgment

Appreciation is expressed to the Japan Society for the Promotion of Science for a Fellowship, March-July 1985, during which this work was completed; Dr. H. Fukuda of the Institute of Interdisciplinary Research, University of Tokyo, for providing the properties of metal matrix composites used herein; Prof. A. Kobayashi and the University of Tokyo for the Guest Professorship in the Institute of Interdisciplinary Research; and Dr. F. A. Kulacki, Professor and Chairman of Mechanical and Aerospace Engineering, and the University of Delaware for permitting acceptance of the Fellowship and Guest Professorship at the University of Tokyo.

References

[1]Vinson, J.R. and Shore, S., "Bibliography on Methods of Structural Optimization for Flat Sandwich Panels," U.S Naval Air Engineering Center, Rept. NAEC-ASL-1082, April 1965.

[2]Vinson, J.R. and Shore, S., "Methods of Structural Optimization for Flat Sandwich Panels," U.S. Naval Air Engineering Center, Rept. NAEC-ASL-1083, April 1965.

[3]Vinson, J.R. and Shore, S., "Design Procedure for Structural Optimization of Flat Sandwich Panels," U.S. Naval Air Engineering Center, Rept. NAEC-ASL-1084, April 1965.

[4]Housner, J.M. and Stein, M., "Numerical Analysis and Parametric Studies of the Buckling of Composite Orthotropic Compression and Shear Panels," NASA TN D-7996, Oct. 1975.

[5]Hirano, Y., "Optimum Design of Laminated Plates under Axial Compression," *AIAA Journal*, Vol. 17, Sept. 1979, pp. 1017-1019.

[6]Hirano, Y., "Stability Optimization of Laminated Composite Plates," Institute of Space and Aeronautical Sciences, Rept. 579, Vol. 45, No. 4, June 1980, pp. 69-85.

[7]Vinson, J.R. and Sierakowski, R.L., *The Behavior of Structures Composed of Composite Materials*, Martinus-Nijhoff, Dordrecht, the Netherlands, 1985.

[8]"Introduction to Metal Matrix Composite Materials," MMIAC Tutorial Series, No. 272, June 1982.

[9]Fukuda, H., Institute of Interdisciplinary Research, University of Tokyo, Japan, personal communication, 1985.

[10]Lagace, P.A., Massachusetts Institute of Technology, Cambridge, personal communication, 1985.

[11]Hojo, M. and Hayashi, R., "Effect of Defects on the Shear Fatigue Properties of CFRP Laminates," IPRI, Tsukuba, Japan, to be published.

From the AIAA Progress in Astronautics and Aeronautics Series...

FUNDAMENTALS OF SOLID-PROPELLANT COMBUSTION – v. 90

Edited by Kenneth K. Kuo, The Pennsylvania State University and Martin Summerfield, Princeton Combustion Research Laboratories, Inc.

In this volume distinguished researchers treat the diverse technical disciplines of solid-propellant combustion in fifteen chapters. Each chapter presents a survey of previous work, detailed theoretical formulations and experimental methods, and experimental and theoretical results, and then interprets technological gaps and research directions. The chapters cover rocket propellants and combustion characteristics; chemistry ignition and combustion of ammonium perchlorate-based propellants; thermal behavior of RDX and HMX; chemistry of nitrate ester and nitramine propellants; solid-propellant ignition theories and experiments; flame spreading and overall ignition transient; steady-state burning of homogeneous propellants and steady-state burning of composite propellants under zero cross-flow situations; experimental observations of combustion instability; theoretical analysis of combustion instability and smokeless propellants.

For years to come, this authoritative and compendious work will be an indispensable tool for combustion scientists, chemists, and chemical engineers concerned with modern propellants, as well as for applied physicists. Its thorough coverage provides necessary background for advanced students.

Published in 1984, 891 pp., 6×9 illus. (some color plates), $59.50 Mem., $89.50 List; ISBN 0-915928-84-1

TO ORDER WRITE: Publications Order Dept., AIAA, 1633 Broadway, New York, N.Y. 10019